DECOMPOSITION-BASED GLOBAL OPTIMIZATION FOR OPTIMAL POWER FLOW PROBLEMS

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Outline

- Motivation
- A power distribution system design problem
- Nonconvex generalized Benders decomposition
- Domain Reduction
- Conclusions
Optimal Power Flow Problems

Optimal Power Flow (OPF) problem is to optimize electric power generation, transmission and distribution networks while satisfying physical system constraints and operation limits.

- Classic economic dispatch problem
- Unit commitment (UC) problem
- Security constrained UC problem
- Capacity planning problem
- Power distribution system design
Steady-State AC Power Flow Equations (in ACOPF)

\[ V_k \rightarrow Y_{km} = Z_{km}^{-1} \rightarrow V_m \]

\[ V_k = |V_k| e^{i\theta_k} = E_k + jF_k \]
\[ V_m = |V_m| e^{i\theta_m} = E_m + jF_m \]
\[ Y_{km} = G_{km} + jB_{km} \]

AC power flow equations:

- Polar form:

\[ P_{km} = |V_k|^2 G_{km} - |V_k||V_m| G_{km} \cos \theta_{km} - |V_k||V_m| B_{km} \sin \theta_{km} \]
\[ Q_{km} = -|V_k|^2 B_{km} + |V_k||V_m| B_{km} \cos \theta_{km} - |V_k||V_m| G_{km} \sin \theta_{km} \]

- Rectangular form:

\[ P_{km} = \left( E_k^2 + F_k^2 \right) G_{km} - \left( E_k E_m + F_k F_m \right) G_{km} + \left( E_k F_m - F_k E_m \right) B_{km} \]
\[ Q_{km} = -\left( E_k^2 + F_k^2 \right) B_{km} + \left( E_k E_m + F_k F_m \right) B_{km} + \left( E_k F_m - F_k E_m \right) G_{km} \]
Approximate Steady-State AC Power Flow Equations (in DCOPF)

\[ V_k Y_{km} = Z_{km}^{-1} V_m \]

\[ V_k = |V_k| e^{j\theta_k} = E_k + jF_k \]
\[ V_m = |V_m| e^{j\theta_m} = E_m + jF_m \]
\[ Y_{km} = G_{km} + jB_{km} \]

\[ P_{km} = |V_k|^2 G_{km} - |V_k||V_m| G_{km} \cos \theta_{km} - |V_k||V_m| B_{km} \sin \theta_{km} \]
\[ Q_{km} = -|V_k|^2 B_{km} + |V_k||V_m| B_{km} \cos \theta_{km} - |V_k||V_m| G_{km} \sin \theta_{km} \]

(a) \( G_{km} \) is negligible
(b) \( \theta_{km} \approx 0 \)
(c) \( V_k \approx V_m \) (can be scaled to 1)

\[ P_{km} = -B_{km} \theta_{km} \]
\[ Q_{km} \approx 0 \]
Solution Methods for OPF Problems
(Frank et al., 2012a/2012b)

- Gradient based methods for NLP formulations

- Branch-and-cut methods for MILP formulations

- Decomposition based methods for large-scale LP/NLP/MILP/MINLP formulations
  (a) Lagrangian decomposition/dual decomposition
  (b) Alternating direction method of multipliers (ADMM)
  (c) Progressive hedging
  (d) Benders decomposition and its extensions

- Meta-heuristic methods

Global optimization is rarely considered.
It may be practical when solution time limit is not relatively tight.
A Power Distribution System Design Problem

The CRBM Medium Office Power Distribution System (Frank and Rebenneck, 2015)

The diagram shows the power distribution system for a medium office building, including the utility connection, feeder distribution, and various load centers. The system is designed to supply power to different areas of the building, such as HVAC systems, lighting, and receptacle loads. The diagram includes specifications such as voltage levels, load capacities, and impedance ratios.
The Problem Structure

Minimize Energy supplied from AC utility over 24 time periods

Subject to

- Constraints on distribution system design decisions
- Operation model for time period 1
  - Activity balances at nodes
  - Voltage-power-current relations at branches, sources, and loads
  - Power relations at AC-DC converters
  - Voltage polar-rectangular representation relations
  - Other constraints (linear)
- Operation model for time period s
A Simplified Representation of the Optimization Problem

\[
\begin{align*}
\text{min} & \quad \text{Linear objective function} \\
\text{s.t.} & \quad V^2 = E^2 + F^2, \quad \text{Voltage magnitude, real and imaginary parts relations} \\
& \quad W^{VV} = V^2, \\
& \quad W^{EE} = E^2, \\
& \quad W^{FF} = F^2, \\
& \quad W^{PP} = P_{\text{sum}}^2, \quad \text{Needed for power relations at converters} \\
& \quad W^{EF} = EF, \quad \text{Needed for voltage-power relations at branches} \\
& \quad \text{Other linear constraints.}
\end{align*}
\]
A Power Distribution System Design Problem

Global Optimization via Convex Relaxation (Frank and Rebenbeck, 2015)

Original problem

\[
\begin{align*}
\min & \quad \text{Linear objective function} \\
\text{s.t.} & \quad V^2 = E^2 + F^2, \\
& \quad W^{VV} = V^2, \\
& \quad W^{EE} = E^2, \\
& \quad W^{FF} = F^2, \\
& \quad W^{PP} = P_{\text{sum}}^2, \\
& \quad W^{EF} = EF, \\
& \quad \text{Other linear constraints.}
\end{align*}
\]

Fixing design decisions

Upper bounding problem

\[
\min \quad y^1
\]

\[
\begin{align*}
\text{s.t.} & \quad W^{VV} = W^{EE} + W^{FF}, \\
& \quad W^{VV} \geq \alpha^V_i V + \beta^V_i, \quad i \in I^{VV}, \\
& \quad W^{VV} \leq (V^U + V^L)V - V^U V^L, \\
& \quad W^{EE} \geq \alpha^E_i E + \beta^E_i, \quad i \in I^{EE}, \\
& \quad W^{EE} \leq (E^U + E^L)E - E^U E^L, \\
& \quad W^{FF} \geq \alpha^F_i F + \beta^F_i, \quad i \in I^{FF}, \\
& \quad W^{FF} \leq (F^U + F^L)F - F^U F^L, \\
& \quad W^{PP} \geq \alpha^P_i P_{\text{sum}} + \beta^P_i, \quad i \in I^{PP}, \\
& \quad W^{PP} \leq (P_{\text{sum}}^U + P_{\text{sum}}^L)P_{\text{sum}} - P_{\text{sum}}^U P_{\text{sum}}^L, \\
& \quad W^{EF} \geq E^L E + F^L F - F^L E^L, \\
& \quad W^{EF} \geq E^U E + F^U F - F^U E^U, \\
& \quad W^{EF} \leq E^L E + F^U F - F^U E^L, \\
& \quad W^{EF} \leq E^U E + F^L F - F^L E^U, \\
& \quad \text{Other linear constraints.}
\end{align*}
\]

Lower bounding problem
A Power Distribution System Design Problem

Computational Results

1. Problem size
   Continuous var: 31920
   Binary var: 48

2. Computing environment
   - 2.93 GHz (single-core) CPU, 48 GB RM, Ubuntu 12.04
   - GAMS 24.1.3, BARON 12.5.0, CPLEX 12.5.1

3. Computational Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Total time (sec)</th>
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Note: [1] BARON did not return a feasible solution after 400000 sec
Nonconvex Generalized Benders Decomposition (NGBD) (Li et al., 2011, Li and Li, 2016)
Nonconvex Generalized Benders Decomposition (NGBD) (Li et al., 2011, Li and Li, 2016)

Original problem

\[
\min_{x_1, \ldots, x_s, y} \sum_{h=1}^{s} f_h(x_h) + c_h^T y \\
\text{s.t.} \quad g_h(x_h) + B_h y \leq 0, \quad h = 1, \ldots, s \\
x_h \in X_h, \quad h = 1, \ldots, s \\
y \in Y
\]

Primal problem 1 \quad \cdots \quad Primal problem s

\[
\begin{align*}
\text{obj}_{pp}^i &= \min_{x_s} f_s(x_s) + c_s^T y^l \\
\text{s.t.} \quad g_s(x_s) + B_s y^l \leq 0, \\
x_s &\in X_s.
\end{align*}
\]

\[
\text{obj}_{pp}^i \leq UB - \sum_{h \neq s} \text{obj}_{BDPP}^h
\]

GBD primal 1 \quad \cdots \quad GBD primal s

\[
\begin{align*}
\text{obj}_{GBDPP}^k &= \min_{x_s} f_s^c(x_s) + c_s^T y^k \\
\text{s.t.} \quad g_s^c(x_s) + B_s y^k \leq 0, \\
x_s &\in X_s^c.
\end{align*}
\]

\[
\begin{align*}
\min_{\eta} \quad & \eta \\
\text{s.t.} \quad & \eta \geq \alpha_i^T y + \beta_i, \quad \forall i \in I^k, \\
& 0 \geq \alpha_j^T y + \beta_j, \quad \forall j \in J^k.
\end{align*}
\]

GBD relaxed master
SOCP Relaxations

\[
\begin{align*}
\text{min} & \quad \text{Linear objective function} \\
\text{s.t.} & \quad V^2 = E^2 + F^2, \\
& \quad W^{VV} = V^2, \\
& \quad W^{EE} = E^2, \\
& \quad W^{FF} = F^2, \\
& \quad W^{PP} = P_{\text{sum}}^2, \\
& \quad W^{EF} = EF, \\
& \quad \text{Other linear constraints.}
\end{align*}
\]

Nonlinear convex evenlopes

\[
\begin{align*}
W^{VV} + 0.5 & \geq \sqrt{V^2 + (W^{VV})^2 + 0.5^2}, \\
W^{EE} + 0.5 & \geq \sqrt{E^2 + (W^{EE})^2 + 0.5^2}, \\
W^{FF} + 0.5 & \geq \sqrt{F^2 + (W^{FF})^2 + 0.5^2}, \\
W^{PP} + 0.5 & \geq \sqrt{P_{\text{sum}}^2 + (W^{PP})^2 + 0.5^2}, \\
W^{VV} & = W^{EE} + W^{FF},
\end{align*}
\]

Linear concave evenlopes

\[
\begin{align*}
W^{VV} & \leq (V^U + V^L)V - V^UV^L, \\
W^{EE} & \leq (E^U + E^L)E - E^UE^L, \\
W^{FF} & \leq (F^U + F^L)F - F^UF^L, \\
W^{PP} & \leq (P_{\text{sum}}^U + P_{\text{sum}}^L)P_{\text{sum}} - P_{\text{sum}}^UP_{\text{sum}}^L, \\
W^{EF} & \geq E^L E + F^L F - F^L E^L, \\
W^{EF} & \geq E^U E + F^U F - F^U E^U, \\
W^{EF} & \leq E^L E + F^U F - F^L E^L, \\
W^{EF} & \leq E^U E + F^L F - F^L E^U, \\
& \quad \text{Other linear constraints.}
\end{align*}
\]
Failure of Constraint Qualification

\[
\begin{align*}
\min \quad & \text{Linear objective function} \\
\text{s.t.} \quad & V \geq \sqrt{E^2 + F^2}, \\
& W_{VV} + 0.5 \geq \sqrt{V^2 + (W_{VV})^2 + 0.5^2}, \\
& W_{EE} + 0.5 \geq \sqrt{E^2 + (W_{EE})^2 + 0.5^2}, \\
& W_{FF} + 0.5 \geq \sqrt{F^2 + (W_{FF})^2 + 0.5^2}, \\
& W_{PP} + 0.5 \geq \sqrt{P_{\text{sum}}^2 + (W_{PP})^2 + 0.5^2}, \\
& W_{VV} = W_{EE} + W_{FF}, \\
& W_{VV} \leq (V^U + V^L)V - V^U V^L, \\
& W_{EE} \leq (E^U + E^L)E - E^U E^L, \\
& W_{FF} \leq (F^U + F^L)F - F^U F^L, \\
& W_{PP} \leq (P_{\text{sum}}^U + P_{\text{sum}}^L)P_{\text{sum}} - P_{\text{sum}}^U P_{\text{sum}}^L, \\
& W_{EF} \geq E^L E + F^L F - F^U E^L, \\
& W_{EF} \geq E^U E + F^U F - F^U E^U, \\
& W_{EF} \leq E^L E + F^U F - F^U E^L, \\
& W_{EF} \leq E^U E + F^L F - F^L E^U, \\
& \text{Other linear constraints.}
\end{align*}
\]

\[
\begin{align*}
\min \quad & \text{Linear objective function} + p \cdot S \\
\text{s.t.} \quad & V + S \geq \sqrt{E^2 + F^2}, \\
& W_{VV} + 0.5 + S \geq \sqrt{V^2 + (W_{VV})^2 + 0.5^2}, \\
& W_{EE} + 0.5 + S \geq \sqrt{E^2 + (W_{EE})^2 + 0.5^2}, \\
& W_{FF} + 0.5 + S \geq \sqrt{F^2 + (W_{FF})^2 + 0.5^2}, \\
& W_{PP} + 0.5 + S \geq \sqrt{P_{\text{sum}}^2 + (W_{PP})^2 + 0.5^2}, \\
& S \geq 0, \\
& W_{VV} = W_{EE} + W_{FF}, \\
& W_{VV} \leq (V^U + V^L)V - V^U V^L, \\
& W_{EE} \leq (E^U + E^L)E - E^U E^L, \\
& W_{FF} \leq (F^U + F^L)F - F^U F^L, \\
& W_{PP} \leq (P_{\text{sum}}^U + P_{\text{sum}}^L)P_{\text{sum}} - P_{\text{sum}}^U P_{\text{sum}}^L, \\
& W_{EF} \geq E^L E + F^L F - F^U E^L, \\
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& W_{EF} \leq E^U E + F^L F - F^L E^U, \\
& \text{Other linear constraints.}
\end{align*}
\]

No Slater point if the branch/bus is not developed.
### Computational Results

1. **Problem size**

   Continuous var: 31920  
   Binary var: 48

2. **Computing environment**

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Note: [1] BARON did not return a feasible solution after 400000 sec  
[2] A 2.40 GHz (rather than 2.93 GHz) CPU is used for all NGBD methods.
Computational Results

Convergence rate of NGBD with different convex relaxations
Domain Reduction for NGBD

Domain Reduction for Tighter Convex Relaxations
Optimization Based Domain Reduction

\[
\begin{align*}
\min_x & \quad f(x) \\
\text{s.t.} & \quad g(x) \leq 0
\end{align*}
\]

As difficult as the original problem!

\[
\begin{align*}
\min_{x_i} & \quad x_i \\
\text{s.t.} & \quad g(x) \leq 0
\end{align*}
\]

\[
\begin{align*}
\min_{x_i} & \quad x_i \\
\text{s.t.} & \quad g^c(x) \leq 0 \\
& \quad UB \geq f^c(x)
\end{align*}
\]
Optimization Based Domain Reduction for NGBD

\[
\begin{align*}
\min_{x_{h,i}} & \quad x_{h,i} \\
\text{s.t.} & \quad g_h^c(x_h) + B_h y \leq 0, \quad h = 1, \ldots, s \\
& \quad x_h \in X_h^c, \quad h = 1, \ldots, s \\
& \quad y \in Y \\
UB & \geq \sum_{h=1}^s f_h^c(x_h) + c_h^T y
\end{align*}
\]

Convex MINLP/MILP on full search space

This optimality cut links all scenarios!

\[
\begin{align*}
\min_{x_h, y} & \quad x_{h,i} \\
\text{s.t.} & \quad g_h^c(x_h) + B_h y \leq 0, \\
& \quad x_h \in X_h^c, \\
& \quad y \in Y, \\
UB & \geq \text{obj}_{GBDPP} + f_{h,j}^L(y, x_h), \quad \forall j \in T^k
\end{align*}
\]

Convex MINLP/MILP on reduced search space

“Scenario optimality cut”

(Li and Li, 2016)
Marginal Based Domain Reduction

Optimization based domain reduction:

\[ x^{BC} = \min_{x_{lo} \leq x \leq x_{up}, u_f(x) \leq UB} x \]

Marginal based domain reduction:

\[ x^{RR} = x^{up} - \frac{(UB - LB)}{\lambda} \]
Marginal based domain reduction:
1. No extra subproblems need to be solved
2. The lower bounding problem has to be convex

In NGBD, the GBD primal problem is a convex lower bounding problem for the NGBD primal problem!
NGBD With Domain Reduction (Li and Li, 2016)
Computational Results

1. Problem size
   Continuous var: 31920
   Binary var: 48

2. Computing environment
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<td>NGBD + DR</td>
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<td>2542</td>
<td>750</td>
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Note: [1] BARON did not return a feasible solution after 400000 sec
      [2] A 2.40 GHz (rather than 2.93 GHz) CPU is used for all NGBD methods.
Conclusions

- Global optimization for offline optimal power flow problems is possible.
- Global optimization can be expedited via exploitation of the problem structure.
- Decomposition-based global optimization can benefit from domain reduction.
- A tighter SOCP relaxation? SDP relaxation?

Acknowledgements

- NSERC discovery
References


Thank you!